Narrowband Oscillations in the Upper Equatorial Ocean. 
Part I: Interpretation as Shear Instabilities 

J. N. MOURM, J. D. NASH, AND W. D. SMYTH

College of Oceanic and Atmospheric Sciences, Oregon State University, Corvallis, Oregon

(Manuscript received June 22, 2010, in final form )

ABSTRACT

Extended measurements of temperature fluctuations that include the turbulence wavenumber band have now been made using rapidly sampled fast thermistors at multiple depths above the core of the Equatorial Undercurrent on the Tropical Atmosphere-Ocean (TAO) mooring at 0°, 140°W. These measurements include the signature of narrowband oscillations as well as turbulence, from which the temperature variance dissipation rate ($\chi_T$) and the turbulence energy dissipation rate ($\epsilon$) are estimated. The narrowband oscillations are characterized by:

- groupiness - packets consist of O(10) oscillations;
- spectral peaks up to two orders of magnitude above background;
- a clear day/night cycle with more intensive activity at night;
- enhanced mixing rates;
- frequencies of $1 - 2 \times 10^{-3}$Hz, close to both the local buoyancy and shear frequencies, $N/2\pi$ and $S/2\pi$, and remarkably constant in time;
- high vertical coherence over at least 30 m scales; and
- abrupt vertical phase change ($\pi/2$ over $< 20$ m).

The abrupt vertical phase change is consistent with instabilities formed in stratified shear flows. Linear stability analysis applied to measured velocity and density profiles leads to predicted frequencies that match those of the observed oscillations. We suggest that the observed oscillation frequencies are set by the phase speed and wavelength of instabilities as opposed to the Doppler-shifting of internal gravity waves with intrinsic frequency set by the local stratification $N$.

1. Introduction

Because of the importance of turbulent heat and momentum transports to the dynamics of the equatorial current system (Crawford and Osborn 1981; McCreary 1981), experiments to examine the nature of mixing in the central equatorial Pacific were conducted in 1984 (TROPIC HEAT), 1987 (TROPIC HEAT 2), 1991 (Tropical Instability Wave Experiment, TIWE) and in autumn 2008. From these experiments, we gained detailed glimpses into the complexity of small-scale processes within the equatorial current system. The intense diurnal cycle of mixing extending beneath the mixed layer was first observed in 1984 (Moum and Caldwell 1985; Gregg et al. 1985) and later found (in 1987) to be associated with intermittently-occurring bursts of oscillations with frequencies near the local buoyancy frequency (McPhaden and Peters 1992) and horizontal wavelengths of 150-250 m (Moum et al. 1992a). Recently, surprisingly intense subsurface mixing was observed above the core of the Equatorial Undercurrent (EUC) in association with tropical instability waves (Moum et al. 2009), and that does not cycle on a daily basis.

There are several ways that narrowband oscillations (frequencies near $N$, 200 m wavelengths) may be generated; candidates include shear instability in the stratified fluid above the EUC core (Sun et al. 1998; Mack and Hebert 1997; Smyth and Moum 2002), shear instabilities formed in the mixed layer when shear is enhanced by easterly wind intensifications (Skyllingstad and Denbo 1994), convectively-driven eddies in the mixed layer (Gregg et al. 1985) and an obstacle effect associated with sheared flow over a perturbed mixed layer base (Wijesekera and Dillon 1991). It is possible that different instability mechanisms occur at different times (or even simulta-
neously). It is important for us to determine exactly how these instabilities occur in order to make the link to the next larger temporal and spatial scales, which is the way we will improve mixing parameterizations.

We have recently demonstrated a new means of obtaining moored measurements of mixing (Moum and Nash 2009) that consists of a pair of fast-response thermistors protruding from a pressure case attached to a mooring cable. The instrument, which we call $\chi$pod, is vaned to steer the thermistors into the flow, thus permitting measurement of undisturbed flow. This has proven successful at current speeds above 0.05 m s$^{-1}$. The technique is especially useful in the upper equatorial current system, where strong currents prevail.

The purpose of these measurements is to obtain records of mixing that are sufficiently long to evaluate the interplay between mixing, waves and the larger circulation scales that affect interannual phenomena such as El Niño and La Niña. Long time series are required because attempts to parameterize mixing from the intensive but short equatorial turbulence profiling experiments to date have not been particularly successful (Zaron and Moum 2009). We suggest this is partly due to the fact that our limited experiments have not measured the full range of equatorial mixing states.

A complementary set of objectives includes obtaining a useful measure of the turbulent heat flux over long time scales to allow estimates of the vertical divergence and thus, heating rates through the water column. In this way, we anticipate determining the role that variations in mixing have on the large-scale equatorial current structure and, particularly, on the sea surface temperature.

Here we examine an 8-month record from four depths above the core of the EUC with the intention of pointing out the variety of states of mixing and how these are related to narrowband oscillations, which are ubiquitous in

![Figure 1](image_url)
these time series. We are hesitant to refer to these a priori as “internal waves”, as we wish to distinguish freely propagating internal gravity waves from shear instability waves, and hence use the term oscillation to cover both phenomena.

We begin with a short description of the data (§ 2) and the background oceanographic conditions during the deployment (§ 3). We then take a closer look at a 52-day portion of the record that most clearly contains the narrowband signal, and examine the temperature variability and properties of the narrowband oscillations (§ 4) and their relationship to turbulence (§ 5). A discussion follows in which we argue that these observations are consistent with shear instabilities, which prompts a linear stability analysis (LSA) using observed currents and stratification, resulting in the finding that the observed peak frequencies of narrowband oscillations are determined by the phase speeds and wavelengths of shear instabilities formed above the EUC core (§ 6). This is followed by a short discussion (§ 7), and a summary and conclusions (§ 8). In the companion paper (Smyth et al. 2010, hereafter Part II), the computational method is presented, the computed instabilities are described in greater detail and a theory is advanced to explain the narrowband character of the oscillations.

2. Details of the deployment

Five χpods were deployed on the TAO mooring at 0°, 140°W on 18 September 2006 at 29, 39, 49, 59 and 84 m depth (χpod depths are shown on the image plots in Fig. 1). These were recovered on 30 May 2007 and a refurbished set of six χpods deployed on the replacement mooring. The unit targeted for 84 m was damaged prior to deployment and no data were recorded. All of the other units provided continuous data for the duration of the deployment. The χpods provided temperature at 10 Hz sample rate and its derivative at 120 Hz. Using currents measured nearby and the cable motion as sensed by linear accelerometers on board the χpods, the flow speed past the sensors is determined. The temperature variance dissipation rate, $\chi_T$, is then computed as described by Moum and Nash (2009). From this, we estimate the turbulence kinetic energy dissipation rate, $\epsilon_T$; and the turbulence diffusion coefficient for heat, $K_T$.

The background temperature structure in the upper 100 m at 0°, 140°W during the deployment was derived from temperature measurements on the TAO mooring at 1, 5, 10, 13, 20, 25, 28, 40, 45, 48, 60, 80 and 100 m (Fig. 1b). From these, an estimate of mixed layer depth was made as the depth at which the temperature deviated from its value at 1 m by 0.015°C (daily maxima of mixed layer depths are shown in Fig. 1b). Wind stress (Fig. 1a) was derived from wind speed measurements on the surface float using bulk formulae (Fairall et al. 1996). A nearby subsurface mooring with 150 kHz narrowband acoustic Doppler current profiler at 270 m depth provided a measure of ambient currents in the upper 250 m (zonal, $u$, meridional, $v$, currents are shown in Fig. 1c,d and squared shear computed from these in Fig. 1e), augmented by discrete measurements on the surface float using bulk formulae (Fairall et al. 1996). The local buoyancy frequency $N$ is computed in two ways. A large-scale estimate is derived from temperature and conductivity measurements on the TAO mooring. The spacing of temperature sensors is given in the previous paragraph. Conductivity sensors were spaced at 20 m intervals in the 20 to 60 m depth range that includes our χpods. These measurements were interpolated to 5 m vertical spacing, density ($\rho$) was computed, and from this $N^2 = -g\rho/\rho$, where $\rho_z$ is the local vertical temperature gradient and $g$ the acceleration due to gravity. For the purpose of this analysis, this estimate of $N$ is used for

\[1\] The subscript $\chi$ is used to indicate that $\epsilon$ is computed from microscale temperature, not shear.
time-averaged, vertical profiles. An estimate of $N$ that is local in both time and space to our $\chi$ pods and does not require differencing independent measurements (an operation that amplifies uncertainties in gradients) is derived directly from $\chi$ pod measurements of $T$. Vertical pumping of the TAO mooring by surface gravity waves acting on the surface float causes the $\chi$ pods to move vertically (nominally, $\pm 1.5$ m) about their mean depth on the mooring (Moum and Nash 2009). $T$ is fit to $z$ over 2-minute intervals from which the local vertical gradient, $T_z$, is derived. From this follows $N_T^2 = g\alpha T_z$, where $\alpha$ is the thermal expansion coefficient of seawater. $N$ and $N_T$ are compared in (§4). Because it turns out that the contribution of salinity gradients are small during the periods of greatest interest, and because $N_T$ provides a local estimate uncompromised by interpolation, we employ this value in combination with wave displacement in (§4) to estimate wave energy. $N_T$ is also used in the $\chi$ pod estimate of $\chi_T$ and $\epsilon_\chi$, as prescribed by Moum and Nash (2009). Again, during the periods of interest, $\chi_T$ and $\epsilon_\chi$ are found not to be biased by our choice of $N$. Uses of $N$ and $N_T$ are clearly distinguished in the figures and text to follow.

3. Background conditions

The variability in Fig. 1 reflects the great range of equatorial motions. Deep mixed layers preceded the sur-
depths followed. Accompanying the surfacing of the EUC is the shallowing of a deeper sustained shear layer above the EUC core into the depth range of the χ pods (denoted in Figs. 1b-e). The temperature record at each of the four χ pods is shown in Fig. 2. The χ pod temperatures indicate good agreement with nearby (1 m offset) temperature sensors on the TAO mooring, typically within a simple offset of 0.1°C.

The frequency at the amplitude maxima in Fig. 3 is identified by finding the peak in each spectrum and requiring it to have a minimum amplitude of $2.3 \times 10^{-3}$ K$^2$. The result for the χ pod at 29 m is shown in Fig. 5a. The short-term variability in $f_{NB}$ is significantly smaller than that in $N_T/2\pi$ and the two frequencies are practically uncorrelated (actually, weakly-correlated in a negative sense, $r=-0.25$).

A comparison between $N$ and $N_T$ is also seen in Fig. 5a. Salinity gradients apparently occur with either sign (interpolated to 29 m in this record; destabilizing early in the record when stratification is weak and stabilizing at the end of the record). However, during the periods of interest for this study (when $f_{NB}$ is well-defined), the effect of salinity gradient on $N^2$ is reasonably small. It is not known how well the estimate of $N^2$ from the coarsely-spaced and slowly-sampled moored sensors relates to local variations relevant to computation of $\chi_T$.

The distinctive peak in temperature spectra suggests that we can fairly reliably extract the narrowband signal by filtering. We define the perturbation temperature signal, $T'$, by highpass filtering the temperature record at $3 \times 10^{-4}$ Hz. A relative measure of the narrowband displacement is then

$$\zeta = T'/T_\epsilon,$$

from which we estimate $N^2 \zeta^2$ as twice the available potential energy associated with the narrowband oscillations, following Moun et al. (1992a).

A two-day example that shows the variability of the narrowband signal and its relationship to the turbulence is represented in Fig. 6. Unfortunately, the solar radiation signal on the TAO mooring was incomplete for this deployment. To indicate the time of the solar day, an estimate of clear sky, short-wave radiation is shown in Fig. 6a, as determined from a simple model\footnote{$f_{SW}^2 = I_0 \sin(\omega_0 (t - t_0))$, when $\sin(\omega_0 (t - t_0)) > 0$, $f_{SW}^2 = 0$, otherwise, where $I_0 = 1100$ W m$^{-2}$, $\omega_0 = 2\pi$ radians/day, $t_0 = 140/360+1/4$ days}. The duration of narrowband bursts is several hours, most intensely at night. These are typically accompanied by high $\epsilon_\chi$. Generally, $\epsilon_\chi$ is lower when narrowband oscillations are absent. In the first energetic wave group (just past midday on 01 January), the oscillations appear at all depths at close to the same time. However, high $\epsilon_\chi$ is observed first at the upper χ pod, and thence progressively later with depth. The other oscillation groups seem to show that both oscillations and turbulence appear at the upper χ pod first, following which they appear progressively later with increasing depth.

During the period of the measurements shown in Fig. 6, the core of the EUC was about 125 m depth (Fig. 7a). The four χ pods were above the EUC core, in the westward-flowing South Equatorial Current (SEC). The spectral peak at $f_{NB} \approx 0.0017$ Hz (Fig. 7d) is similar to the local value of $N/2\pi$ (6 cph) at 40 - 70 m (Fig. 7b).
Fig. 5. a) Time series of $N_T/2\pi$ (black line), $f_N$ (symbols) and $N/2\pi$ (gray line) at 29 m. $f_N$ corresponds to the spectral peaks in (Fig. 3c). b) $N_T/2\pi$ vs $f_N$.

Fig. 6. a) Incoming shortwave radiation as determined from a clear sky solar radiation model, b) $\zeta$ (1 minute means), and c) $\epsilon_\chi$ (1 minute averages of 1 s spectral estimates) computed from the four $\chi$ pods for the first two days of January 2007. In c), the dotted baselines represent $\epsilon_\chi = 10^{-9}$ m$^2$s$^{-3}$. In this and all plots, times are UTC.
but $f_{NB} > N/2\pi$ at 29 m. Oscillations are vertically coherent at all measurement depths (Fig. 7e), with a strong vertical phase dependence (Fig. 7f). The phase lag at 39 m is approximately $\pi/4$ relative to 29 m, and $\pi/2$ at 49 and 59 m (also relative to 29 m), suggesting that the $\pi/2$ phase shift occurs over a distance of $\Delta z < 20$ m. These results are similar to those derived from towed thermistor chain measurements nearby and along the equator in 1987 (Moum et al. 1992a).

The detailed structure of the narrowband oscillations is diverse. Five 12-h examples from the period shown in Fig. 3 are reproduced in Fig. 8 (each example is denoted by a Roman numeral at left in Fig. 8, which is echoed in Fig. 3). Over this time period, the mixed layer surfaced from about 70 m to 35 m (Fig. 1). The zero-crossing point of the zonal velocity (which separates the EUC from the SEC) surfaced from about 50 m to 15 m and the zero-crossing point of $\epsilon_X$ is approximately $\pi/4$ at 29 m. Oscillations are vertically coherent over the full depth range (Fig. 7f). The phase lag at 39 m is approximately $\pi/4$ relative to 29 m, and $\pi/2$ at 49 and 59 m (also relative to 29 m), suggesting that the $\pi/2$ phase shift occurs over a distance of $\Delta z < 20$ m. These results are similar to those derived from towed thermistor chain measurements nearby and along the equator in 1987 (Moum et al. 1992a).

These examples were chosen because each is distinguished by a narrowband spectrum but with unique characteristics that we note in turn.

I The $\chi$ pods were all located in the westward-flowing SEC. At 39-59 m, $f_{NB} \approx N/2\pi$, and at 29 m, $f_{NB} > N/2\pi$. For reference, the spectrum at 59 m (Fig. 8c) is reproduced in grey in Fig. 8d. The narrowband signal is most intense in the first half of the record, with oscillations gradually growing in amplitude over the first 12 cycles, and then gradually decreasing in amplitude over the final 12 cycles. This is accompanied by a gradual increase and decrease in $\epsilon_X$.

II The zero-crossing point of the zonal velocity has surfaced to the depth of the third $\chi$ pod so that the upper 2 $\chi$ pods are in westward-flowing water and the lowest in eastward-flowing water. $f_{NB} < N/2\pi$ at all of the $\chi$ pod depths. The dominant signal near the middle of the record consists of oscillations growing in amplitude over approximately 12 cycles, after which they vanish. This abrupt ending is accompanied by an equally abrupt and large increase in $\epsilon$ at the depths of the lowest 3 $\chi$ pods.

III $f_{NB}$ is close to that in I, II. As in II, there is a significantly greater broadband energy at $f < f_{NB}$ (Fig. 8IId) than there is in I.

IV This particularly strong event with $f_{NB} > N/2\pi$ at 29 and 39 m, has the signature of the time-inverse of II. That is, initially large amplitude oscillations decrease in amplitude over about 12 cycles. High $\epsilon_X$ accompanies the leading, largest oscillations.

V In this example of low frequency oscillations, $f_{NB} < N/2\pi$ at all depths.

5. Relationship to mixing

Two trends are apparent in the 52-day record (20 December 2006 - 10 February 2007) during which narrowband waves were consistently observed (Fig. 9a). Firstly, there is a clear diurnal cycle in both $\epsilon_X$ and $N^2\zeta^2$ (Fig. 9b,c). There is also a modulation of both $\epsilon_X$ and $N^2\zeta^2$ on the time scales of the narrowband waves seen in (Fig. 9a). The day / night differences in $N^2\zeta^2$ can be seen in histograms of 1 h values sorted into daytime and nighttime periods (Fig. 10). These differences are most pronounced nearest the sea surface.

Due to covariation on both diurnal and longer time scales, $N^2\zeta^2$ is highly correlated to $\epsilon_X$ ($r = 0.64$, 95% confidence limits [0.60, 0.69], Fig. 11). For comparison, the data points plotted in Moum et al. (1992a), Fig. 9 are shown in red ($r = 0.83$). The current $\chi$ pod estimates overlap the parameter space of the 1987 measurements and extend it.

Lines of unity slope in Fig. 11 represent constant time scales related to the decay of energy $N^2\zeta^2$ by turbulence dissipation, $\tau_d \approx N^2\zeta^2/\epsilon$. These suggest that the narrowband oscillations decay rapidly, on a time scale of O(1 h), or a few oscillation periods. This would seem to be consistent with the relatively few oscillations that comprise a group as well as the patchiness of the narrowband signal.

6. Mechanistic interpretation

The narrowband signal that we have described from these measurements has several distinctive characteristics. The dominant frequency, $f_{NB} \approx 1 - 2 \times 10^{-3}$ Hz, varies remarkably slowly in time (Fig. 3) and is similar in magnitude to both $N/2\pi$ and $S/2\pi$. At times, $f_{NB} > N/2\pi$, although it is not generally observed to be greater than the maximum value of $N$ above the EUC core (Fig. 8).

The oscillations typically appear in groups of about 10 before disappearing from the time series (Fig. 8). The groups are associated with elevated mixing (Fig. 11). Packets appear as both symmetric groups with large amplitudes and most-elevated mixing in the middle of the group (Fig. 8, group I) and as highly asymmetric groups, with largest amplitudes and most-elevated mixing at either trailing or leading edge (Fig. 8, groups II, IV). If we interpret the oscillations as propagating wave packets, it is impossible to distinguish trailing from leading edge in the time series data.

Signals are vertically coherent over the full depth range of the measurements (29-59 m) and exhibit a change in phase of $\pi/2$ over a vertical span $\leq 20$ m. Both of these characteristics were observed in the towed thermistor chain measurements of Moum et al. (1992a), while the high coherence was detected in the moored observations of McPhaden and Peters (1992). Because of a lack of confidence in relative timing between sensors, McPhaden and Peters (1992) were unable to determine whether ver-
tectical differences in phase at the frequencies of the narrowband signal differed from 0.

In an introduction to the original set of papers describing the narrowband phenomenon beneath the equatorial mixed layer, Moum et al. (1992b) made two significant points. They acknowledged the difficulty in making the distinction between freely-propagating internal gravity waves and forced shear instabilities from the observations. And they recognized the importance of doing so, primarily because of the potential for vertical redistribution of momentum by vertically-propagating internal waves that would not exist in the case of short-lived shear instabilities.

A number of hypotheses have been proposed to explain this narrowband signal beneath the equatorial mixed layer in the form of internal waves. Wijesekera and Dillon (1991) suggested that the interaction of turbulent overturns in the nighttime mixed layer with the mixed layer base in the highly sheared equatorial current system might produce an internal wave field. From two-dimensional simulations, Skyllingstad and Denbo (1994) showed generation of internal waves in the equatorial surface layer during periods of easterly wind intensifications by shear instabilities generated there. For the data presented here, the shear is generated by easterly winds. In fact, our data show instances of enhance-

ment of narrowband wave activity when easterly winds increase (compare Fig. 3a and 3c). However, there are enough counterexamples to indicate that if easterly wind intensification is a factor, it does not work in isolation. More recently, Pham and Sarkar (2010) have used three-dimensional simulations to show the generation and growth of internal waves by energetic vortex tubes ejected from Kelvin-Helmholtz instabilities into the sheared fluid above the core of the EUC.

At the same time, arguments have been made that the narrowband signal is directly related to shear instabilities. LSA using observed profiles of currents and density (Sun et al. 1998) has shown the generation of shear instabilities with wavelengths close to those observed by Moum et al. (1992a). Most relevant may be the observation of the $\sim \pi/2$ change in phase across the waves, which echoes the results of Moum et al. (1992a) and is expected for growing Kelvin-Helmholtz instabilities (Smyth and Peltier 1989; Baines and Mitsudera 1994). We proceed with this argument by applying LSA to the present data and comparing to the properties of these observations.

a. Comparison with textbook shear instability

Three rules of thumb arise from studies of the idealized shear instability case of the Kelvin-Helmholtz form
Fig. 8. a) Averaged zonal ($u$; blue) and meridional ($v$; red) velocity profiles. (b) Averaged buoyancy frequency ($N/2\pi$; black) and shear ($S/2\pi$; red) profiles. Averages were computed over the 12 h time periods indicated in (c), which shows temperature at 29 (red), 39 (cyan), 49 (blue), and 59 m (black) depths from the $\chi$-pods, following the color convention introduced in Fig. 2. (d) $\epsilon_x$. (e) Temperature spectra normalized by the mean value of $N$ at that depth. The grey line reproduced in all of the plots is the 59 m spectral line from the first (upper) time period. The grey bars represent the peaks in the spectra, and are reproduced in (b).
The observed $\pi/2$ phase shift (Fig. 7f) is consistent with property #2 above. The thickness of the layer over which this phase shift occurs provides an estimate for the thickness of the shear layer that generated the instability. In this example, the thickness is $\leq 20$ m. For the purpose of rough scaling, we use 20 m. If property #1 above is valid, the resulting wavelength would be 7 times this, or $\lambda \approx 140$ m, consistent with previous observations (Moum et al. 1992a).

The phase velocity is likely to be negative as the $\pi/2$ phase change of the disturbance occurs over the depth range of the $\chi$ pods, which is within the westward-flowing SEC (Fig. 7a,b). To estimate this phase velocity, we use the observed frequency, $f_{NB} = 1.7 \times 10^{-3}$ Hz (Fig. 7d). As defined above, $f_{NB}$ is an absolute value; the actual frequency is negative. We therefore compute the phase velocity as $c = -\lambda f_{NB} = -0.24$ m s$^{-1}$. This is within the range of the background velocity (Fig. 7a) in agreement with the semicircle theorem (Howard 1961). It is, however, significantly slower than the background current in the depth range of the $\chi$ pods where the phase change is observed, contrary to property #3 above. This discrepancy is addressed in section § 6b.

In summary, the observed oscillations are consistent with at least some of the standard properties of Kelvin-Helmholtz instability on a shear layer.

b. LSA of observed mean states
To expand on this result, we have carried out stability analyses of hourly-averaged profiles of velocity and stratification. The linear stability problem for a viscous, diffusive, stratified, parallel shear flow was solved in matrix form, as described in Part II.

Vertical profiles of current shear and stratification, $N^2$, were computed from the relatively coarsely-resolved measurements available from the TAO mooring (Fig. 1). Under-resolution of both shear and stratification means that $Ri$ is likely overestimated, as high-resolution measurements show small $Ri$ on vertical scales smaller than the 5 m vertical samples and time scales shorter than 1 hour samples available (Moum et al. 2009). We assume that the hourly-averaged profiles represent a stationary, parallel flow perturbed by infinitesimal disturbances, whereas in fact the flow is continuously subjected to large-amplitude perturbations. Despite these caveats, the normal mode spectrum yields a testable prediction of the fastest-growing disturbance.

The LSA yields growth rates and phase speeds as functions of wavenumber. The mode corresponding to the wavenumber where the growth rate is a maximum is the one most likely to reach observable amplitude. The predicted frequency is $f = c/\lambda$, where $c$ is the phase speed of the fastest-growing mode.

![Fig. 10](image)

**Fig. 10.** Histograms of $N^2 \zeta^2$ at a) 29 m, b) 39 m, c) 49 m and d) 59 m depths. These were sorted into daytime (1800-0200 UTC) and nighttime periods (0600-1400), which are represented by light and dark grey shading, respectively.

![Fig. 11](image)

**Fig. 11.** Comparison of $N^2 \zeta^2$ and $\epsilon$ at 29 m based on 1 h averages over the 52-day period shown in Fig 9 - yellow/red points plotted represent the two-dimensional histogram; redder points occur more frequently. The black dots represent estimates from measurements in 1987 of $\epsilon$ using the turbulence profiler, CHAMELEON, and a thermistor chain towed behind the ship (Moum et al. 1992a). The correlation for the $\chi$pod data is noted at upper left, with 95% confidence limits. The correlation for the 1987 data is 0.83.

![Fig. 12](image)

**Fig. 12.** Amplitude (a) and phase (b) of the vertical displacement eigenfunction corresponding to the 69 fastest-growing modes with frequencies near to the peak frequency in Fig. 9d, as determined from a LSA using observed currents and stratification from TAO mooring data at 0$^\circ$, 140$^\circ$W. Amplitude is nondimensional and is defined up to an arbitrary scaling factor. The vertical axis is depth referenced to the critical layer at which the fastest-growing mode was found, and nondimensionalized by its wavelength. Horizontal dashed lines identify a layer of thickness $\lambda/7$ about $z_c$. Thick lines are mean values, thin lines encompass 50% of the eigenfunctions.

This analysis was repeated for all hourly-averaged profiles during the time period shown in Figs. 3 and 9 and modes were selected according to criteria given in Part II. The sum of these analyses delivered an ensemble of 155 instability events. It was usually not possible to cor-
relate a specific instability with an observed increase in narrowband signal. This is not surprising for several reasons. First of all, the aforementioned low resolution of the input data. Secondly, the velocity and density measurements are not colocated. The temperature and conductivity measurements used to compute the stratification were obtained from the TAO surface mooring which was deployed typically at least 1 km from the subsurface mooring that houses the acoustic Doppler current profiler from which velocity profiles were obtained. Lateral inhomogeneities in currents and stratification between these two locations contribute to both false positive and false negative indications of instabilities. Thirdly, any observed instability must have had sufficient time to grow to observable amplitude. During this time, it will have advected at speed c, which is typically \( c \neq 0 \), meaning that the instability was formed elsewhere. Again, lateral inhomogeneities in currents and stratification will contribute to lack of agreement between observed and predicted instabilities. In view of these unavoidable limitations, we focus not on individual events but rather on the statistical properties of the ensemble.

Properties of the 155 unstable modes are discussed in detail in Part II and briefly reviewed here. Growth rates range from 1 to 11 e-foldings per hour (for comparison, a close look at Fig. 6b suggests that the growth rate of the first large packet at mid-day on 01 January 2007 was approximately 2 e-foldings per hour). As expected from the dominance of zonal shear, unstable modes were generally zonally-propagating, with phase velocities ranging from \(-1 \text{ m s}^{-1}\) in the SEC to \(+1 \text{ m s}^{-1}\) in the EUC. Negative values were considerably more common, but values close to the peak SEC velocity were rare because modes with critical levels above 37.5 m depth (where the quality of the TAO measurements was diminished) were rejected. Critical level depths extended nearly to the depth of the EUC core. Although unstable modes could in theory have wavelengths up to 7 times the depth of the EUC core, or 700 m, computed modes all had \( \lambda \leq 340 \text{ m} \).

The computed range of wavelengths is consistent with existing observations (Moum et al. 1992a; Lien et al. 1996). For the purpose of direct comparison of the properties of observed and predicted instabilities, we compare:

- \( f_{NB} \) with the apparent frequency \( f = c/\lambda \) due to the advection of computed instabilities past our observation point;
- the vertical phase dependence of the computed instabilities with those observed; and
- predicted wavelengths to estimated shear layer thicknesses, to assess consistency with rule-of-thumb #1.

1) FREQUENCIES OF SHEAR INSTABILITIES

The ensemble of frequencies determined for the 155 instabilities over the 52-day record shown in Fig. 9a is represented by the histogram in Fig. 9d. Despite the fact that the instabilities arise from a range of critical depths \( (z_c) \) above the core of the EUC, encompassing a range of both \( c \) and \( \lambda \) (Part II), the peak frequency of the distribution is very close to that defined by the spectral peak at frequency \( f_{NB} = 1.7 \times 10^{-3} \text{ Hz} \) (Fig. 9d). Peak frequencies of the observations and computed unstable modes are both close in magnitude to \( N/2\pi \) (Fig. 5a) although \( f_{NB} \) is not temporally correlated with \( N \) (Fig. 5b).

2) VERTICAL PHASE DEPENDENCE

To illustrate the general vertical structure of the instabilities, we computed composite amplitude and phase profiles as follows. We compiled vertical displacement eigenfunctions for 69 unstable modes selected by the requirement that \( 10^{-3} < |f| < 10^{-2.7} \text{ Hz} \), representing the frequency range of the narrowband instabilities (indicated by the mode of the histogram in Fig. 9d). Each vertical displacement eigenfunction was normalized by its value at the critical level. The eigenfunctions were then converted to polar form, and the amplitude and the tangent of the phase were sorted into bins according to the scaled vertical coordinate \( (z - z_c)/\lambda \). The median and quartiles were computed for each bin.

Results are shown in Fig. 12. The amplitude peak clearly coincides with \( z_c \) in Fig 12a, and the phase changes by close to \( \pi/2 \) radians across the shear layer, in accord with our rule-of-thumb #2 and the observed vertical phase shift (Fig. 7f). Closer inspection of Fig 12b reveals that the phase shift actually occurs not over the whole shear layer but over approximately the inner half of that layer. We will show that this property resolves the phase speed discrepancy noted in § 6a.

3) RELATIONSHIP BETWEEN WAVELENGTHS AND SHEAR LAYER THICKNESSES

The shear layer thickness \( (h_s) \) was determined for each mode by fitting the shear magnitude to a function \( S_N \text{sech}^2[(z - z_c)/h_s] \), where \( S_N \) is the shear at the critical level. Fig. 13 shows statistics of the shear layer thickness and the wavelength for modes with frequencies in the range \( 10^{-3} < |f| < 10^{-2.7} \). The median shear layer thickness is 30 m. Combined with our previous observation that the phase shift occurs over half the shear layer, the phase shift should typically occur over 15 m in accordance with the example shown in Fig. 7.

The median wavelength for this subsample of the unstable modes is 245 m, consistent with the observations of Moum et al. (1992a). If we now recompute the phase velocity of the mode shown in Fig. 7 using this wavelength, we find \( c = -0.42 \text{ m s}^{-1} \). This is roughly consistent with the background flow speeds observed in the depth range of the \( \chi \) pods. This improved agreement with linear theory is thanks mainly to the refinement of our previous assumption that the phase shift occupies the entire shear layer.

\[ \chi = \int_{-\infty}^{\infty} \frac{\text{sech}^2}{\text{sech}^2 + 1} \]
The canonical model for shear layer instability is the Kelvin-Helmholtz form, derived from hyperbolic-tangent profiles of velocity and density (Hazel 1972), whence comes the expectation that the wavelength is about 7 times the shear layer thickness. In our computed modes, the mean value of $\lambda / h_s$ is 7.4, the median value is 8.8 and 75% of the values of $\lambda / h_s$ lie within the range [6,11] (Fig. 13). While it is likely that this ratio is altered by the range of naturally-occurring shear profiles that spawn unstable modes (Part II), this result suggests that the Kelvin-Helmholtz model provides a reasonable description of the observed instabilities.

In sum, the apparent frequencies of predicted shear instabilities are close to $f_{NB}$ (and close to $N/2\pi$ as well). 92% of the computed wavelengths are within a factor of 2 of $7h_s$. And the predicted vertical phase dependence of $\pi/2$ across $h_s$ echoes the observed phase shifts.

7. Discussion

The LSA does not produce a preference for nighttime over daytime instabilities. While this might be considered to be inconsistent with our interpretation of the narrowband signal (which has a strong daily variation, greater nearest the surface, as indicated by Fig. 10) as shear instabilities, it may also be due to the relatively coarse quality of the data used to perform the LSA.

One of the great benefits of the TAO data is the long, freely-available time series from which we can test theories such as ours. At the same time, we are pushing that data beyond its intended use as it is not resolved finely enough, either in time or in space, to adequately capture the true variability of shear and stratification in equatorial flows. This can be clearly seen by comparison to the richness of small-scale variability shown in the measurements of Moum et al. (2009). Perhaps of most concern is the lack of high-quality data near the surface where the downward migration of the mixed layer at night includes both the highly-stratified and highly-sheared mixed layer base, which is a potential instability trigger on a daily time scale. Yet despite the limitations of the data, we have found that instabilities are predicted and that the properties of these instabilities are consistent with the properties of the observed narrowband oscillations. In a complementary future analysis, we will examine the well-resolved measurements described by Moum et al. (2009) to determine if LSA can define the daily variations.

A detailed one-to-one comparison of observed and predicted instabilities is beyond the capability of the data available from the TAO moorings (which do not offer coincident velocity and density profiles). It may also be generally impossible simply due to the fact that practically all instabilities (those characterized by $c \neq 0$) formed in sheared flows such as these are not observed until they have achieved observable amplitude. Hence, observed shear instabilities must coincide with mean flow conditions different from those in which they were created.

8. Summary and conclusions

These measurements provide a depiction of the narrowband oscillations at the equator that has not previously been available. They demonstrate the following characteristics.

- The oscillations are groupy. They appear as groups of $O(10)$ individual oscillations (Fig. 8).
- Spectral peaks at $f_{NB}$ are approximately two orders of magnitude greater than at background frequencies, even when computed over two-day periods.
- $f_{NB}$ is similar in magnitude to $N/2\pi$ but these are not correlated in time.
- Daily variation in oscillation potential energy is greatest at night and concentrated at the surface (Fig. 10).
• Mixing is enhanced (Fig. 11).
• There is high vertical coherence at $f_{NB}$.
• Finally, there exists strong vertical phase dependence at $f_{NB}$, typified by the abrupt $\pi/2$ phase shift in Fig. 7f.

At minimum, the observed abrupt vertical phase change motivates our interpretation of these oscillations as shear instabilities. Together with previous analyses (Sun et al. 1998; Smyth and Moum 2002) that have shown instability wavelengths comparable to observed wavelengths (Moum et al. 1992b), this has prompted the computation of the properties of shear instabilities from the available TAO data. From this analysis, we have shown that the localized shear layers where instabilities are formed are approximately $\lambda/7$ in thickness, with apparent frequencies comparable to $f_{NB}$ (figure 9d), further supporting the case that the observed narrowband signal is due to shear instability resembling the classic Kelvin-Helmholz form. This scenario differs from that of a Doppler-shifted wave field suggested by McPhaden and Peters (1992).

These measurements support previous observations demonstrating the short-lived nature of packets of narrowband oscillations formed at night in the upper equatorial thermocline (McPhaden and Peters 1992; Moum et al. 1992a). The ratio $N^2/\zeta^2/\epsilon_s$ may be considered a time scale representing the decay of the energy in the oscillations solely due to turbulence. This time scale is O(1 h) (Fig. 11), and corresponds roughly to the observed duration of packets of O(10 periods) at 5 cph (in the reference frame of the fixed observer). The suggestion has been that the combination of space-time patchiness and high turbulence means these packets are generated locally and decay locally (Moum et al. 1992a). Our analysis indicates that the properties of these oscillations are consistent with shear instabilities formed from the hourly mean structure of currents and stratification.

It seems that the observed narrowband signal in the temperature record is consistent with locally-generated shear instabilities that do not propagate or advect very far before breaking. The fact that $f_{NB}$ is near to $N/2\pi$ has to do with advection of the spatial structure of the instability characterized by $\lambda$ at $c$ past the measurement point, rather than oscillation of a gravity wave at frequency $N$ (Part II). In Part II, we show that $N$ is a natural scale because (1) the dominant frequency of an ensemble of shear instabilities scales with $S$, and (2) $S, N$ are closely related in this near-critical $Ri$ flow.

The strong variability in turbulence associated with that of these narrowband oscillations contributes to large variations in vertical mixing. It is important that we sample the full range of conditions in order to determine the complete role of mixing in the equatorial heat balance. The $\chi$ pod measurements on TAO moorings are continuing with this objective.

Acknowledgements. This work was funded by the National Science Foundation (0424133, 0622922, 0728375). Mike Neeley-Brown and Ray Kreth were responsible for construction of the $\chi$ pods. Alexander Perlin helped to organize and process the data. Numerous NOAA personnel have aided this effort with their professional handling of our instruments on deployment and recovery. We are indebted to NOAA’s TAO Project Office for continued support in $\chi$ pod deployments and recoveries and for the use of the mooring TAO data from 0°140°W.

REFERENCES


